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# Comparative Analysis of ARMA and GARMA Models in Predicting the Blooming Dynamics of Rafflesia arnoldii in Bengkulu Province

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**Abstract**

Bengkulu Province, known as the Land of Rafflesia, is home to Rafflesia arnoldii, a rare and iconic flowering species. Understanding the temporal dynamics of its blooming frequency is essential not only for effective conservation planning but also for strengthening flora-based ecotourism initiatives. Time series forecasting has been widely applied to ecological and environmental data, with the Autoregressive Moving Average (ARMA) model being one of the most commonly used approaches. However, ARMA relies on the white-noise assumption, which is often violated in count data such as the number of Rafflesia arnoldii blooms, leading to reduced accuracy. To address this limitation, this study applies the Generalized Autoregressive Moving Average (GARMA) model, which accommodates non-Gaussian data from the exponential family, including Poisson and Negative Binomial distributions. The dataset consists of monthly records of blooming events collected by the Bengkulu Natural Resources Conservation Agency from January 2015 to December 2023. Model performance was evaluated using Root Mean Square Error (RMSE), Mean Absolute Deviation (MAD), and Mean Arctangent Absolute Percentage Error (MAAPE). Results show that GARMA(1,0) achieved RMSE = 2.424, MAD = 2.224, and MAAPE = 31%, while GARMA(0,2) achieved RMSE = 2.550, MAD = 1.483, and MAAPE = 26%. In contrast, ARMA(1,0) performed less effectively, with RMSE = 3.694, MAD = 2.676, and MAAPE = 36%. These findings demonstrate that GARMA provides more stable and accurate forecasts, effectively capturing the stochastic properties of count data without depending on residual normality. The study highlights the methodological superiority of GARMA over ARMA, offering both theoretical contributions to time series modeling and practical benefits for biodiversity conservation. By enabling more reliable predictions of Rafflesia arnoldii blooms, GARMA can inform conservation policies and support sustainable tourism strategies in Bengkulu Province.

**Keywords:** ARMA Model; Biodiversity Conservation; Count Data Forecasting; GARMA Model; Rafflesia arnoldii

## I. Introduction

Time series analysis has been extensively applied to model and forecast temporal phenomena across various fields, including ecology and environmental sciences. By exploiting temporal dependence among observations, time series methods provide a systematic framework for predicting future events based on historical data [1] [2]. Among classical approaches, the Autoregressive Moving Average

(ARMA) and the Autoregressive Integrated Moving Average (ARIMA) are widely used due to their parsimonious structure and strong theoretical foundation for modeling stationary processes [1].

To address the limitations of Gaussian-based time series models, the Generalized Autoregressive Moving Average (GARMA) model was introduced as an extension of the classical ARMA framework for non-Gaussian time series [3]. GARMA allows the response variable to follow distributions from the exponential family, such as Poisson or Negative Binomial distributions, making it particularly suitable for modeling count data [4]. Previous studies have shown that GARMA provides improved performance in modeling low-count or overdispersed time series, especially in ecological, epidemiological, and environmental applications where the assumptions of ARMA-type models are violated [5].

In ecological and environmental studies, time series observations frequently take the form of count data, representing the number of events occurring within a fixed time interval. Such data are discrete, non-negative, and often exhibit skewness and overdispersion [4] [6]. When modeled using conventional ARMA-type approaches, these characteristics may lead to inefficient parameter estimation and unreliable forecasts, particularly in cases involving rare or intermittent events [5].

In contrast, ARMA and ARIMA models assume that the error terms form a Gaussian white-noise process characterized by independence, constant variance, and normality [7]. This assumption is frequently unrealistic for ecological count data, especially when modeling rare species, where observations are sparse and highly variable. Consequently, there remains a lack of empirical evidence quantifying the extent to which GARMA outperforms ARMA in forecasting ecological count time series.

Rafflesia arnoldii is a rare parasitic flowering plant and an ecological icon of Bengkulu Province, Indonesia. Its blooming events occur infrequently and irregularly, making the associated time series particularly challenging to model using conventional approaches. Despite the growing use of GARMA models for non-Gaussian time series, few studies have explicitly compared the forecasting performance of ARMA and GARMA for ecological count data, particularly in the context of rare endemic species such as Rafflesia arnoldii. To the best of the authors' knowledge, no prior study has systematically evaluated these two modeling frameworks using long-term bloom count data of Rafflesia arnoldii.

The novelty of this study lies in its systematic comparative evaluation of ARMA and GARMA models applied to a rare-species ecological time series. From a theoretical perspective, this study contributes empirical evidence regarding the limitations of Gaussian-based ARMA models and demonstrates the advantages of GARMA in capturing the stochastic structure of non-Gaussian count data. From a practical perspective, the study shows how GARMA-based forecasts can improve prediction accuracy for rare ecological events, offering valuable insights for biodiversity conservation, field monitoring, and flora-based ecotourism planning. These contributions underscore the relevance of the present study for readers interested in ecological time series modeling and applied conservation analytics.

## II. Methodology

### *A. Data Collection and Preparation*

In this study, the data used are secondary records on the number of Rafflesia arnoldii blooms in Bengkulu Province. The dataset consists of monthly observations from January 2015 to December 2023, covering a total of 108 months, obtained from the Bengkulu Natural Resources Conservation Agency (BKSDA). Descriptive statistics were first computed to provide an initial overview of the data characteristics. The dataset was subsequently partitioned into training and testing subsets, with 93% of the earliest observations (100 months) used for model training and the remaining 7% of the most recent observations (8 months) reserved for out-of-sample testing. Although this split differs from the commonly used 70-80% training proportion, it was deliberately chosen due to the limited sample size and the temporal nature of the data. In ecological time series involving rare events, preserving a sufficiently long training period is essential for reliable estimation of autoregressive and moving average structures, while the testing set must remain chronologically ordered to avoid information leakage. The

chosen split therefore reflects a trade-off between parameter estimation stability and meaningful forecast evaluation at the end of the observation period.

Before model identification, the training data were examined to ensure compliance with the stationarity assumption. Preliminary autoregressive and moving average orders were identified based on the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots. Parameter estimation for the ARMA model was conducted using the Maximum Likelihood Estimation (MLE) method, followed by significance testing of the estimated parameters and verification of the white-noise assumption. In parallel, the GARMA model was constructed and its parameters were estimated under the assumption that the response variable follows a distribution from the exponential family.

All statistical analyses and model estimations were carried out using the R statistical computing environment. The ARMA models were estimated using standard time series functions available in the stats package, while the GARMA models were fitted using specialized routines for generalized time series modeling. Diagnostic tests, including the Augmented Dickey-Fuller test, Ljung-Box test, and Kolmogorov-Smirnov test, were also implemented within R. Forecasting accuracy was evaluated using Root Mean Square Error (RMSE), Mean Absolute Deviation (MAD), and Mean Arctangent Absolute Percentage Error (MAAPE).

### *B. Stationarity and the Augmented Dickey-Fuller Test*

Stationarity is a fundamental concept in time series analysis because the validity and reliability of many statistical models depend on this property. In general, stationarity can be divided into two types: mean stationarity and variance stationarity. Mean stationarity refers to a condition in which the time series fluctuates around a constant average value throughout the observation period, while variance stationarity requires that the variability of the data remains constant over time. In this study, only mean stationarity is considered, while variance stationarity is disregarded. Identifying mean stationarity is essential, as non-stationary data may produce spurious regression results, misleading parameter estimates, and ultimately unreliable forecasts. Ensuring stationarity allows the underlying stochastic process to be treated as stable, thereby making the model estimations more robust and the forecasts more accurate [7].

To evaluate mean stationarity, unit root tests are typically employed. One of the most widely used approaches is the Augmented Dickey-Fuller (ADF) test, introduced and popularized by David Dickey and Wayne Fuller. The ADF test examines whether the time series contains a unit root, with the null hypothesis stating that the series is non-stationary ($H_0: \beta = 0$) and the alternative hypothesis indicating that the series is stationary ($H_1: \beta \neq 0$). The test statistic is computed as

$$\tau = \frac{\hat{\beta}}{SE(\hat{\beta})}, \tag{1}$$

where $\hat{\beta}$ is the least squares estimator of $\beta$, and $SE(\hat{\beta})$ is its standard error. If the calculated $\tau$ is smaller than the critical value provided in the Dickey-Fuller table, or if the p-value is less than the chosen significance level, the null hypothesis is rejected, suggesting that the data are stationary in mean. Conversely, if these conditions are not met, the null hypothesis cannot be rejected, and the data are deemed non-stationary.

The identification of stationarity, particularly in ecological and biological data such as the blooming frequency of Rafflesia arnoldii, is of great importance. Ecological time series often exhibit trends, seasonality, or random fluctuations that may violate the stationarity assumption. By ensuring that the data are stationary, the researcher can apply time series models such as ARMA or GARMA with greater confidence, leading to parameter estimates that truly reflect the underlying dynamics of the process. Moreover, accurate detection of stationarity directly improves the reliability of forecasting results, which in turn provides more meaningful insights for conservation strategies and policy-making.

### *C. Autoregressive Moving Average (ARMA) Model*

The ARMA model is a widely used time series forecasting approach that assumes past observations influence the current value of the series, which is characterized by two components: the autoregressive

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part AR($p$), which captures the linear dependence between the current observation and its $p$ preceding values, and the moving average part MA($q$), which represents the linear relationship between the current observation and the past $q$ error terms. The ARMA method is appropriate for stationary time series data, where the statistical properties such as mean and variance remain constant over time [8]. The general form of the ARMA($p$,$q$) model is expressed as [9]:

$$Z_t = \phi_0 + \phi_1 Z_{t-1} + \cdots + \phi_p Z_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q}, \quad (2)$$

where $Z_t$ is the observed value at time $t$, in this case representing the number of Rafflesia arnoldii blooms in Bengkulu Province; $\phi_0$ is a constant term; $\phi_p$ denotes the $p$-th autoregressive parameter, which measures the influence of past observations on the present value; $\theta_q$ denotes the $q$-th moving average parameter, which measures the impact of past random shocks (errors) on the current value; and $\varepsilon_t$ is the white noise error term at time $t$, assumed to be independently and identically distributed with zero mean and constant variance.

The ARMA model has several advantages. It is mathematically elegant, relatively simple to estimate, and provides a good balance between model parsimony and predictive accuracy for stationary time series. It is particularly effective when the data exhibit short-term correlations but no significant trends or seasonal patterns. Moreover, ARMA models form the foundation for more advanced forecasting techniques, such as ARIMA and GARMA. However, the ARMA model also has limitations. Its application is restricted to stationary time series, which means that data exhibiting strong trends, seasonality, or structural breaks must first be transformed, often through differencing or detrending. Additionally, ARMA assumes that residuals are uncorrelated and normally distributed, an assumption that is often violated in count data or data from ecological and environmental processes. When these assumptions are not satisfied, the forecasts produced by ARMA may be biased or inefficient, motivating the use of extended models such as GARMA for non-Gaussian count data.

### *D. Generalized Autoregressive Moving Average (GARMA) Model*

The GARMA model is an extension of the classical ARMA framework designed for time series data that do not follow the Gaussian distribution. While ARMA assumes normally distributed residuals and is therefore limited to continuous stationary data, many real-world datasets; particularly in ecological, epidemiological, and environmental studies, consist of count data or other types of non-Gaussian distributions. To address this limitation, Benjamin, Rigby, and Stasinopoulos (1998) developed the GARMA model, which generalizes ARMA to accommodate time series from the exponential family of distributions, including Poisson, Negative Binomial, and Binomial [10].

This flexibility makes GARMA more suitable for modeling discrete data such as the number of Rafflesia arnoldii blooms, where the normality assumption of ARMA is not satisfied. The GARMA($p$,$q$) model can be expressed as follows [3] [11]:

$$\ln(\mu_t) = \beta_0 + \sum_{j=1}^{p} \phi_j \{\ln Z^*_{t-j}\} + \sum_{j=1}^{q} \theta_j \{\ln(Z^*_{t-j}) - \ln(\mu_{t-j})\}, \quad (3)$$

where $Z^*_{t-j} = \max(Z_{t-j}, c)$ with $0 < c \leq 1$. This adjustment ensures that zero values in the time series are replaced by a small constant $c$, thereby avoiding computational issues with the logarithmic transformation. Here, $\mu_t$ represents the conditional mean of the process at time $t$, $\beta_0$ is the regression intercept parameter, $\phi_j$ denotes the autoregressive parameters, and $\theta_j$ represents the moving average parameters. The link function $\ln(\mu_t)$ connects the ARMA structure to the mean parameter of the assumed distribution, enabling GARMA to capture the dynamics of non-Gaussian time series.

In practice, parameter estimation in GARMA models is carried out using MLE, which accommodates the distributional assumptions of the exponential family. Beyond the core parameters namely the intercept $\beta_0$, the autoregressive coefficients $\phi_j$, and the moving average coefficients $\theta_j$, additional terms frequently appear in estimation outputs. For example, the parameter $u_1$ often functions as a scale factor that reflects the dispersion of the assumed distribution, while $fd_1$ represents a fractional differencing parameter that may be included to capture potential long-memory behavior in the series. Likewise, terms

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such as AR(1), MA(1), and MA(2) correspond to estimated autoregressive and moving average coefficients at lags one and two, in line with ARMA notation. These additional parameters are not part of the canonical GARMA equation but emerge in practical software implementations that extend the GARMA framework with fractional differencing or distributional scaling. Including them in the reporting process enhances transparency and facilitates reproducibility of the analysis.

The GARMA model offers several notable advantages over ARMA. By allowing the response variable to follow distributions from the exponential family, GARMA is particularly well-suited for modeling count data and overdispersed ecological outcomes. This flexibility enhances both the interpretability and accuracy of forecasts in contexts where ARMA assumptions are violated. At the same time, GARMA retains the autoregressive and moving average structures of ARMA, thereby preserving the ability to capture temporal dependence while accounting for the discrete nature of the data. Nevertheless, GARMA also has certain limitations. Its parameter estimation is computationally more demanding than ARMA, often requiring iterative numerical methods. Furthermore, the accuracy of GARMA forecasts is highly sensitive to the choice of distributional family, such as Poisson or Negative Binomial, and misspecification can lead to biased results. Despite these challenges, GARMA represents a powerful and flexible alternative to ARMA for analyzing non-Gaussian time series, particularly in ecological applications such as modeling the blooming frequency of Rafflesia arnoldii.

### *E. Testing the White-Noise Assumption*

Validating the white-noise property of residuals is a crucial diagnostic step in time series modeling. A model is considered adequate only if its residuals behave like white noise, meaning that they are normally distributed with zero mean, constant variance, and are independent across time. When these conditions are satisfied, it can be concluded that the model has successfully explained the systematic structure of the data, leaving only random fluctuations unexplained. Conversely, if the residuals exhibit correlation or deviate from normality, the model may be misspecified and its forecasts unreliable.

Residual independence is commonly evaluated using the Ljung-Box test [9]. This test examines whether a group of residual autocorrelation coefficients, up to lag $K$, are jointly equal to zero. The null hypothesis states that residuals are uncorrelated ($H_0: \rho_1 = \rho_2 = \cdots = \rho_k = 0$), while the alternative hypothesis indicates that at least one autocorrelation coefficient is significantly different from zero. The Ljung-Box statistic is defined as:

$$Q = n(n+2)\sum_{k=1}^{K} \frac{(\hat{\rho}_k)^2}{n-k}, n > k, \tag{4}$$

where $n$ is the number of observations, $\hat{\rho}_k$ is the sample autocorrelation of residuals at lag $k$, and $K$ is the maximum lag considered. Under the null hypothesis, $Q$ approximately follows a chi-square distribution with $K$ degrees of freedom. If the test statistic exceeds the chi-square critical value, or if the p-value is less than the chosen significance level $\alpha$, the null hypothesis is rejected, indicating that the residuals are correlated and the white-noise assumption is not satisfied.

In addition to independence, residuals must also follow a normal distribution. This can be assessed using the Kolmogorov-Smirnov (K-S) test [12], which compares the empirical cumulative distribution function (ECDF), denoted as $F_n(x)$, with the theoretical cumulative distribution function (CDF) of a normal distribution, denoted as $F_0(x)$. The hypotheses are defined as: $H_0: F(x) = F_0(x)$, for all $x$, meaning that the residuals follow a normal distribution vs $H_1: F(x) \neq F_0(x)$ for some $x$, meaning that the residuals do not follow a normal distribution. The K-S test statistic is expressed as:

$$D = \sup\{F_n(x) - F_0(x)\}, \tag{5}$$

where $F_n(x)$ is the ECDF of the residuals and $F_0(x)$ is the theoretical CDF of the normal distribution. The statistic $D$ represents the maximum absolute difference between the two distributions. If $D$ exceeds the critical value from the K-S distribution, or if the p-value is smaller than $\alpha$, the null hypothesis is rejected, indicating that the residuals are not normally distributed. Conversely, if $D$ is less than the

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critical value, the null hypothesis cannot be rejected, suggesting that the residuals are consistent with a normal distribution.

By combining the Ljung–Box test for independence and the Kolmogorov-Smirnov test for normality, a comprehensive validation of the white-noise assumption can be achieved. In the context of forecasting the blooming of Rafflesia arnoldii, confirming that residuals are both uncorrelated and normally distributed ensures that the ARMA and GARMA models are properly specified. This validation enhances the statistical reliability of the models and increases confidence in the accuracy and stability of their ecological forecasting applications.

### *F. Measures of Forecast Accuracy*

In forecasting, evaluating the accuracy of predictions is crucial to ensure that the model provides reliable results. Several statistical measures are commonly used to assess forecast accuracy, including Root Mean Square Error (RMSE), Mean Absolute Deviation (MAD), and Mean Arctan Absolute Percentage Error (MAAPE). Each of these metrics offers a different perspective on model performance, helping analysts and decision-makers understand the strengths and limitations of their forecasts.

RMSE is one of the most widely used measures for assessing forecasting accuracy. It calculates the square root of the average squared differences between the predicted and actual values. By giving higher weight to larger errors, RMSE is particularly sensitive to outliers, making it useful when large deviations are especially undesirable. Importantly, RMSE does not have a standardized minimum value; the smaller the RMSE, the closer the model's predictions are to actual observations. The formula for calculating RMSE is as follows [13]:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(Z_t - \hat{Z}_t\right)^2}, \tag{6}$$

where $n$ is the total number of observations, $Z_t$ is the actual number of Rafflesia arnoldii flowers blooming in period t in Bengkulu Province, and $\hat{Z}_t$ is the forecasted value for the same period. RMSE provides a clear way to quantify overall forecast error and is particularly useful when comparing the predictive accuracy of different models, with lower values indicating better performance.

MAD measures the average magnitude of forecast errors without considering their direction, focusing only on the absolute differences [14]. It is calculated as:

$$MAD = \frac{\sum_{t=1}^{n}|Z_t - \hat{Z}_t|}{n}, \tag{7}$$

MAD provides an intuitive understanding of the typical size of forecast errors, with lower values indicating better model performance.

MAAPE is used to evaluate forecast accuracy while addressing some limitations of the traditional Mean Absolute Percentage Error (MAPE), particularly when actual values are very small or close to zero [15]. It is calculated as:

$$MAAPE = \frac{1}{n}\sum_{i=1}^{n}\arctan\left(\left|\frac{Z_t - \hat{Z}_t}{Z_t}\right|\right). \tag{8}$$

Lower MAAPE values indicate a model with better predictive capability, offering a robust alternative to traditional percentage-based error metrics.

## III. Results and Discussion

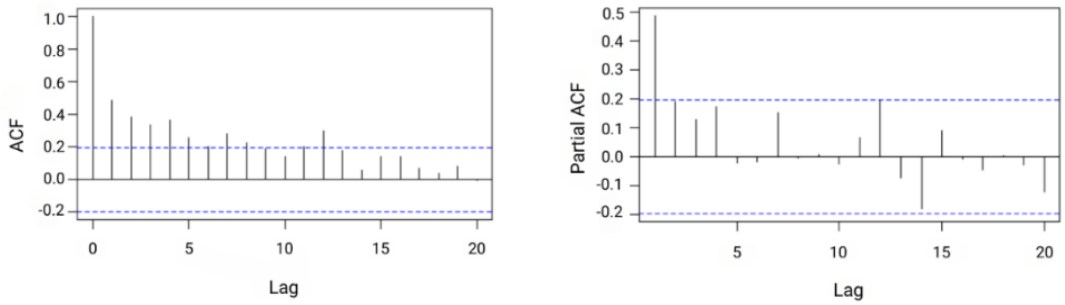
Before identifying the GARMA($p$,$q$) model, a preliminary examination was conducted to ensure that the data on the number of Rafflesia arnoldii flowers blooming in Bengkulu Province followed a distribution from the exponential family. This step is crucial because the GARMA model assumes that the underlying data distribution belongs to the exponential family, which influences both model selection and parameter estimation. To evaluate this assumption, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values were compared across three candidate distributions, as shown in Table 1.

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Table 1. Distribution Test for Rafflesia arnoldii Data

| Distribution | Information Criteria | |
|---|---|---|
| | AIC | BIC |
| Poisson | 504.168 | 506.850 |
| Negative Binomial | 468.308 | 473.673 |
| Geometric | 476.030 | 478.712 |

As observed in Table 1, the Negative Binomial distribution exhibits the smallest AIC and BIC values among the three candidate distributions. This indicates that it provides the best fit for the observed data, capturing both the mean and the overdispersion typically present in count data like the number of blooming Rafflesia flowers. Therefore, it can be concluded that the data on Rafflesia arnoldii blooms in Bengkulu Province follow an exponential family distribution, specifically a Negative Binomial distribution.




(a) (b)

Figure 1. Training Data of Rafflesia arnoldii Blooms with (a) ACF Plot and (b) PACF Plot.

Following the distribution assessment, the dataset was partitioned into 93% training data and 7% testing data, enabling both model fitting and subsequent validation. Before fitting the GARMA model, it was necessary to examine whether the time series of blooming flowers ($Z_t$) is stationary in mean. Mean stationarity ensures that the statistical properties of the series do not change over time, which is a key requirement for ARMA-based modeling. The ADF test was employed for this purpose. The test yielded an ADF statistic of -3.540 with a p-value of 0.042, which is smaller than the significance level $\alpha = 0.05$. This result leads to the rejection of the null hypothesis $H_0$, confirming that the training data are stationary in mean. Establishing stationarity in mean allows for reliable identification of ARMA components.

Figure 1 presents the training data of the number of blooming Rafflesia arnoldii flowers along with the corresponding ACF and PACF plots. By observing these plots, the lags that cross or exceed the confidence interval lines were identified as significant, providing guidance for selecting the appropriate AR($p$) and MA($q$) orders in the ARMA model. These patterns in the ACF and PACF allowed for the determination of a candidate model structure that reflects the temporal dependencies in the bloom data. After selecting the ARMA model structure, the parameters were estimated using the MLE method, which optimizes the likelihood function to yield the most probable parameter values given the observed data. This step, combined with prior assessments of distribution and stationarity, ensures that the resulting GARMA model is statistically valid and capable of accurately capturing the dynamic behavior of Rafflesia arnoldii blooming patterns in Bengkulu Province.

### *A. ARMA Model Parameter Estimation, Assumption Testing, and Practical Implications*

Table 2 presents the parameter estimates of several ARMA models, including ARMA(1,0), ARMA(0,1), ARMA(0,2), ARMA(1,1), and ARMA(1,2). For each model, the intercept and autoregressive (AR) or moving average (MA) coefficients were estimated along with their standard

errors, t-values, and p-values. A parameter was considered statistically significant if the p-value was less than 0.05.

Table 2. ARMA Model Parameter Estimation

| Models | Parameter | Estimate | Std.*Error* | t-value | p-value | Conclusion |
|---|---|---|---|---|---|---|
| ARMA(1,0) | Intercept | 2.589 | 0.383 | 6.757 | 0.000 | Significant |
| | AR(1) | 0.488 | 0.087 | 5.607 | 0.000 | Significant |
| ARMA(0,1) | Intercept | 2.605 | 0.280 | 9.307 | 0.000 | Significant |
| | MA(1) | 0.353 | 0.077 | 4.616 | 0.000 | Significant |
| ARMA(0,2) | Intercept | 2.595 | 0.327 | 7.942 | 0.000 | Significant |
| | MA(1) | 0.416 | 0.100 | 4.154 | 0.000 | Significant |
| | MA(2) | 0.222 | 0.084 | 2.642 | 0.008 | Significant |
| ARMA(1,1) | Intercept | 2.588 | 0.884 | 2.929 | 0.003 | Significant |
| | AR(1) | 0.943 | 0.062 | 15.293 | 0.000 | Significant |
| | MA(1) | -0.698 | 0.133 | -5.244 | 0.000 | Significant |
| ARMA(1,2) | Intercept | 2.568 | 1.023 | 2.510 | 0.012 | Significant |
| | AR(1) | 0.964 | 0.047 | 20.664 | 0.000 | Significant |
| | MA(1) | -0.656 | 0.113 | -5.792 | 0.000 | Significant |
| | MA(2) | -0.106 | 0.106 | -1.003 | 0.316 | Not Significant |

From this analysis, four models, namely ARMA(1,0), ARMA(0,1), ARMA(0,2), and ARMA(1,1), were found to have all significant parameters, indicating that the corresponding coefficients meaningfully contribute to explaining the temporal patterns in the bloom data. In contrast, ARMA(1,2) had one insignificant parameter MA(2), suggesting that including this term may not improve the model fit. After parameter estimation, the white noise assumption of the residuals was tested to assess the adequacy of the models for forecasting.

Table 3. White Noise Diagnostics of ARMA Models

| Model | Normality Test | | Correlation Test | | Conclusion |
|---|---|---|---|---|---|
| | *D* | p-value | *Q* | p-value | |
| ARMA(1,0) | 0.271 | 0.000 | 0.945 | 0.331 | Not normal, Not correlated |
| ARMA(0,1) | 0.233 | 0.000 | 1.107 | 0.293 | Not normal, Not correlated |
| ARMA(0,2) | 0.242 | 0.000 | 0.037 | 0.848 | Not normal, Not correlated |
| ARMA(1,1) | 0.154 | 0.017 | 0.292 | 0.589 | Not normal, Not correlated |

The normality of residuals was evaluated using the Kolmogorov-Smirnov test, as shown in Table 3. All four candidate ARMA models violated the normality assumption (p-values < 0.05), indicating that the residuals were not normally distributed. Next, the residual correlation was examined using the Ljung-Box test (Table 3), which tests whether residuals are uncorrelated over time. The results indicated that residuals of all four models were not significantly correlated (p-values > 0.05), satisfying the independence condition.

Despite residuals being uncorrelated, the failure to meet the normality requirement implies that these ARMA models do not fully satisfy the white noise assumption. Models that fail this assumption are generally not reliable for forecasting, as the error structure may bias predictions. Therefore, the Generalized ARMA (GARMA) model is proposed as an alternative, which extends the ARMA framework to better accommodate non-normal and overdispersed count data such as the number of blooming Rafflesia arnoldii flowers.

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*B. GARMA Model Parameter Estimation and Diagnostic Testing*

The GARMA models are constructed based on the ARMA($p$,$q$) identification results, leading to candidate models GARMA(1,0), GARMA(0,1), GARMA(0,2), and GARMA(1,1). Parameter estimation for these GARMA models will be carried out using the MLE method, which optimizes the likelihood function to produce the most probable parameter values given the observed data.

Table 4. Parameter Estimation of GARMA Models

| GARMA Model | Parameter | Estimate | Std.*Error* | t-value | p-value | Conclusion |
|---|---|---|---|---|---|---|
| GARMA(1,0) | Intercept | 2.610 | 0.001 | 2863.096 | 0.000 | Significant |
| | $u_1$ | *0.999* | *0.157* | *6.366* | *0.000* | *Significant* |
| | $fd_1$ | 0.193 | 0.149 | 1.293 | 0.196 | Not significant |
| | AR(1) | -0.058 | 0.001 | -63.339 | 0.000 | Significant |
| GARMA(0,1) | Intercept | 2.610 | 0.001 | 2848.147 | 0.000 | Significant |
| | $u_1$ | *0.999* | *0.157* | *6.366* | *0.000* | *Significant* |
| | $fd_1$ | 0.201 | 0.201 | 0.999 | 0.318 | Not significant |
| | MA(1) | -0.083 | 0.001 | -91.105 | 0.000 | Significant |
| GARMA(0,2) | Intercept | 2.610 | 0.001 | 2864.252 | 0.000 | Significant |
| | $u_1$ | *0.999* | *0.157* | *6.366* | *0.000* | *Significant* |
| | $fd_1$ | 0.253 | 0.273 | 0.925 | 0.355 | Not significant |
| | MA(1) | -0.189 | 0.126 | -1.493 | 0.136 | Not significant |
| | MA(2) | -0.086 | 0.001 | -94.271 | 0.000 | Significant |
| GARMA(1,1) | Intercept | 2.610 | 0.001 | 2874.635 | 0.000 | Significant |
| | $u_1$ | 0.999 | 0.157 | 6.366 | 0.000 | *Significant* |
| | $fd_1$ | 0.267 | 0.330 | 0.808 | 0.419 | Not significant |
| | AR(1) | 0.345 | 0.432 | 0.799 | 0.424 | Not significant |
| | MA(1) | -0.565 | 0.001 | -622.516 | 0.000 | Significant |

Table 4 presents the parameter estimates for several GARMA models, including GARMA(1,0), GARMA(0,1), GARMA(0,2), and GARMA(1,1). Each model was estimated using MLE, with results reported in terms of coefficients, standard errors, t-values, and p-values.

For the GARMA(1,0) model, the intercept (2.610), $u_1$ (0.999), and AR(1) (-0.058) were statistically significant (p-value < 0.05), while the fractional differencing parameter $fd_1$ was not significant (p-value = 0.196). Similarly, in the GARMA(0,1) model, the intercept, $u_1$, and MA(1) were significant, whereas $fd_1$ was not. The GARMA(0,2) model produced significant results for the intercept, $u_1$, and MA(2), but $fd_1$ and MA(1) were not significant. Lastly, in the GARMA(1,1) model, the intercept, $u_1$, and MA(1) were significant, while $fd_1$ and AR(1) were not.

These findings suggest that while several parameters within each model are significant, no GARMA specification achieved full significance across all estimated parameters. This indicates some instability in parameter contribution, particularly with fractional differencing ($fd_1$) and certain AR/MA terms. In practice, this means that although GARMA captures key temporal features of the bloom data, not all parameters provide meaningful explanatory power.

Table 5. White Noise Diagnostics of GARMA Models

| Model | Normality Test | | Correlation Test | | Conclusion |
|---|---|---|---|---|---|
| | *D* | p-value | *Q* | p-value | |
| GARMA(1,0) | 0.178 | 0.004 | 0.028 | 0.866 | Not normal, Not correlated |
| GARMA(0,1) | 0.528 | 0.000 | 9.467 | 0.002 | Not normal, Correlated |

| Model | Normality Test | | Correlation Test | | Conclusion |
|---|---|---|---|---|---|
| | *D* | p-value | *Q* | p-value | |
| GARMA(0,2) | 0.559 | 0.000 | 2.886 | 0.089 | Not normal, Not correlated |
| GARMA(1,1) | 0.341 | 0.000 | 58.398 | 0.000 | Not normal, Correlated |

The white noise assumption of residuals was assessed through two complementary tests: the Kolmogorov-Smirnov (KS) test for normality and the Ljung-Box test for residual correlation. As shown in Table 5, all four GARMA models failed the normality test (p-value < 0.05), indicating that residual distributions deviate from normality. Such deviations are common in ecological count data, where overdispersion and skewness often occur. The Ljung-Box test provided additional insight into the independence of residuals. Residuals from GARMA(1,0) and GARMA(0,2) were found to be uncorrelated, thus meeting the independence assumption. By contrast, GARMA(0,1) and GARMA(1,1) showed significant autocorrelation, violating this condition.

Taken together, these results indicate that although none of the GARMA models fully satisfy the normality assumption, GARMA(1,0) and GARMA(0,2) emerge as the most reliable candidates because their residuals are independent. This independence is critical, as it minimizes the risk of biased forecasts and enhances model stability. From a practical perspective, employing GARMA(1,0) and GARMA(0,2) for forecasting the blooming of Rafflesia arnoldii flowers provides valuable tools for ecological monitoring and conservation planning. These models can help anticipate flowering patterns more effectively, which in turn supports field scheduling, conservation strategies, and a deeper understanding of the ecological processes governing Rafflesia arnoldii blooms in Bengkulu Province.

### *C. Forecast Accuracy Evaluation*

To further assess model performance, forecast accuracy was evaluated in order to determine whether GARMA($p$,$q$) models provide superior predictive capability compared to the ARMA($p$,$q$) models. Three complementary metrics were employed: Root Mean Square Error (RMSE), Mean Absolute Deviation (MAD), and Mean Arctangent Absolute Percentage Error (MAAPE). RMSE is particularly sensitive to large deviations, penalizing models that produce extreme forecast errors. MAD provides a more balanced measure of average error size, while MAAPE addresses the limitations of traditional percentage-based metrics by offering a bounded and more interpretable error scale, especially useful in ecological datasets where values may be small or highly variable.

Table 6. Forecast Accuracy Metrics Using 7% Testing Data

| Model | | RMSE | MAD | MAAPE |
|---|---|---|---|---|
| ARMA | ARMA(1,0) | 3.694 | 2.676 | 0.367 |
| | ARMA(0,2) | 3.718 | 2.697 | 0.369 |
| GARMA | GARMA(1,0) | 2.424 | 2.224 | 0.316 |
| | GARMA(0,2) | 2.550 | 1.483 | 0.262 |

The results in Table 6 clearly demonstrate the superior performance of GARMA models over their ARMA counterparts. For example, GARMA(1,0) achieved an RMSE of 2.424, significantly lower than ARMA(1,0)'s 3.694, indicating that its forecasts are less prone to large deviations. Similarly, its MAD of 2.224 was smaller than 2.676 for ARMA(1,0), confirming that on average, the forecasts deviate less from the actual observations. The MAAPE value for GARMA(1,0) was 0.316 (≈31%), while ARMA(1,0) yielded 0.367 (≈36%), showing a substantial relative improvement in forecast precision. A similar trend is seen in GARMA(0,2), where RMSE, MAD, and MAAPE values (2.550, 1.483, and 0.262 respectively) were consistently lower than those of ARMA(0,2) (3.718, 2.697, and 0.369).

These improvements are not merely statistical; they carry important practical implications. The reduction in RMSE suggests that GARMA models are better at avoiding large forecasting errors, which is critical when predicting ecological events such as Rafflesia arnoldii blooming, where mistimed

forecasts could result in missed observation opportunities. Lower MAD indicates that GARMA provides forecasts with more consistent accuracy, which is valuable for planning field surveys and allocating conservation resources efficiently. Meanwhile, smaller MAAPE values show that the relative error is substantially reduced, improving the reliability of predictions even when the number of blooms is relatively small.

Overall, both ARMA and GARMA can be considered useful forecasting tools, but GARMA(1,0) and GARMA(0,2) clearly outperform ARMA(1,0) and ARMA(0,2). The consistent reduction in all three error measures indicates that GARMA captures the dynamics of the blooming process more effectively. From a conservation standpoint, adopting GARMA models can enhance the ability of researchers and policymakers to anticipate flowering events, optimize monitoring schedules, and develop proactive conservation strategies for the endangered Rafflesia arnoldii in Bengkulu Province.

Beyond statistical performance, the results also provide important biological and ecological insights into the blooming dynamics of Rafflesia arnoldii. The superior performance of the GARMA(0,2) model indicates that the flowering process of R. arnoldii is better represented as a stochastic count process influenced by short-term random shocks rather than by strong dependence on preceding bloom counts. Ecologically, the moving average structure of order two suggests that blooming events are affected by unobserved environmental disturbances occurring over the previous one to two periods, such as short-term climatic variability, host plant conditions, or localized habitat factors. This finding implies that R. arnoldii does not follow a regular or deterministic flowering cycle, but instead responds sensitively to short-term ecological fluctuations. Consequently, the GARMA(0,2) model captures the irregular and sporadic nature of bloom occurrences more effectively than Gaussian-based ARMA models. From a conservation perspective, this insight highlights the importance of continuous monitoring and adaptive management strategies, as blooming events may occur unpredictably in response to transient environmental conditions.

## IV. Conclusion

Bengkulu Province, widely recognized as the Land of Rafflesia, is home to Rafflesia arnoldii, a rare and iconic flowering species. Understanding the temporal dynamics of its blooming frequency is essential not only for effective conservation planning but also for strengthening flora-based ecotourism initiatives. This study compared ARMA and GARMA models in forecasting the number of R. arnoldii blooms in Bengkulu Province using monthly data from January 2015 to December 2023, obtained from the Bengkulu Natural Resources Conservation Agency (BKSDA).

The results demonstrated that the ARMA model failed to satisfy the white-noise assumption, making it less suitable for modeling count data. In contrast, the GARMA model, which generalizes ARMA to accommodate non-Gaussian data from the exponential family, delivered more reliable forecasts. Specifically, GARMA(1,0) achieved RMSE = 2.424, MAD = 2.224, and MAAPE = 31%, while GARMA(0,2) achieved RMSE = 2.550, MAD = 1.483, and MAAPE = 26%. These error measures were consistently lower than those of ARMA(1,0) and ARMA(0,2), confirming that GARMA is more accurate in capturing the stochastic properties of count data.

Overall, the findings underscore the methodological superiority of GARMA over ARMA in forecasting the blooming frequency of R. arnoldii. Beyond its theoretical contribution to time series modeling, GARMA offers practical benefits by enabling more stable and accurate forecasts. Such improvements in predictive accuracy can inform biodiversity conservation strategies and support sustainable tourism initiatives that emphasize Bengkulu's identity as the Land of Rafflesia. Future research may integrate GARMA with machine learning or hybrid frameworks, or apply it to other ecological and environmental time series, thereby broadening its relevance for both scientific advancement and policy development.

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